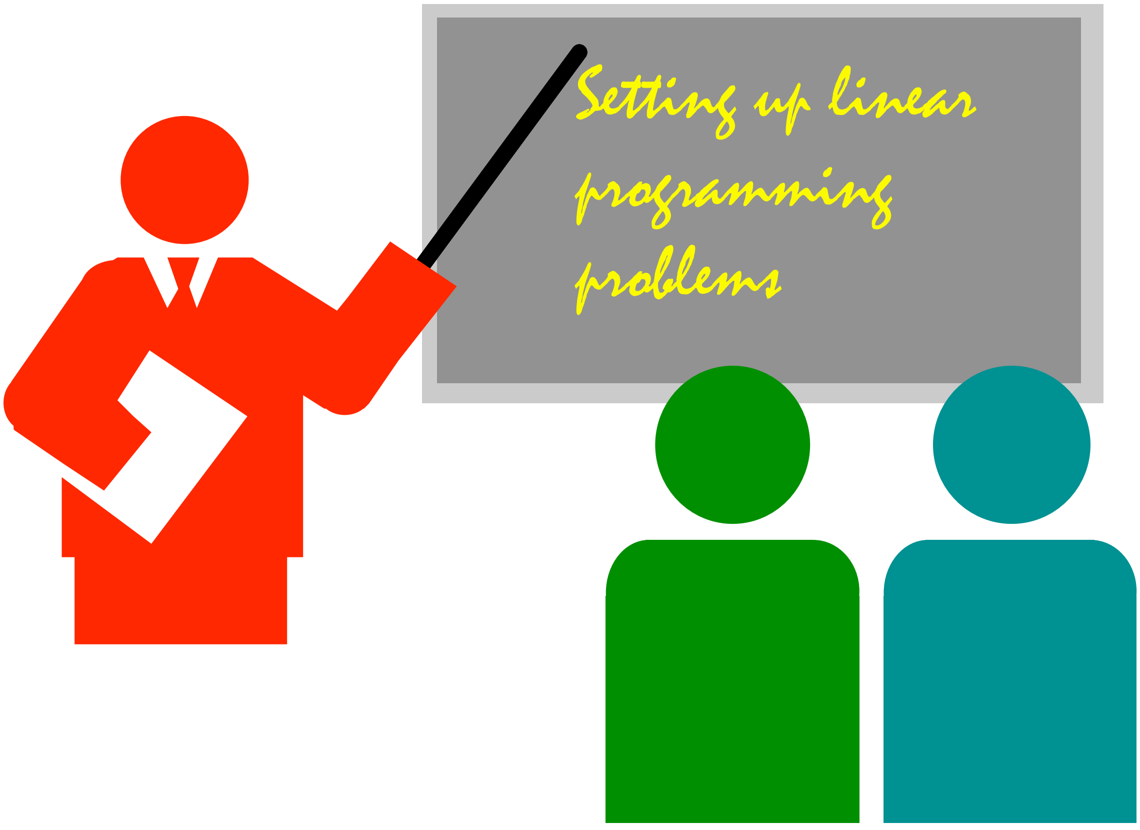
Setting up linear
programming
problems

1. What variables or unknowns are involved?
2. What quantity is to be maximized or minimized and how do I express that quantity in terms of my unknowns?
3. What constraints do I have? How can I express those constraints in terms of my unknowns? (In linear programming problems, this step results in a set of linear inequalities.)

Example - potter making cups and plates

A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 3/4 lb. of clay and each plate uses one lb. of clay. She has 20 hours available for making the cups and plates and has 250 lbs. of clay on hand. She makes a profit of $2 on each cup and $1.50 on each plate. How many cups and how many plates should she make in order to maximize her profit?

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$$6x + 3y \le 1200$$

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$$.75x + y \leq 250$$

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Non-negative constraints: $x \geq 0$, $y \geq 0$

Summary:

x = number of cups the potter makes

y = number of plates the potter makes

The potter wants to maximize profit

$$P = \$2x + \$1.50y$$

Constraints:

$$6x + 3y \le 1200$$

$$.75x + y \le 250$$

$$x \ge 0$$

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Step 1

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Step 2

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Step 3

Farmer planting corn and soybeans

A farmer has a 320 acre farm on which she plants two crops: corn and soybeans. For each acre of corn planted, her expenses are $50 and for each acre of soybeans planted, her expenses are $100. Each acre of corn requires 100 bushels of storage and yields a profit of $60; each acre of soybeans requires 40 bushels of storage and yields a profit of $90. If the total amount of storage space available is 19,200 bushels and the farmer has only $20,000 on hand, how many acres of each crop should she plant in order to maximize her profit? What will her profit be if she follows this strategy?

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land constraint: $x + y \le 320$

$ constraint: $50x + 100y \le 20{,}000$

storage constraint: $100x + 40y \le 19{,}200$

non-negative constraints: $x \ge 0, y \ge 0$

Example - Aluminum and Copper Wire

A plant makes aluminum and copper wire. Each pound of aluminum wire requires 5 kwh of electricity and 1/4 hr. of labor. Each pound of copper wire requires 2 kwh of electricity and 1/2 hr. of labor. Production of copper wire is restricted by the fact that raw materials are available to produce at most 60 lbs./day. Electricity is limited to 500 kwh/day and labor to 40 person−hrs./day. If the profit from aluminum wire is $.25/lb. and the profit from copper is $.40/lb., how much of each should be produced to maximize profit and what is the maximum profit?

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Constraints:

$y \leq 60$

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Constraints:

$$y \leq 60$$

$$5x + 2y \leq 500$$

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Constraints:

$$y \le 60$$

$$5x + 2y \le 500$$

$$.25x + .5y \le 40$$

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Maximize $P = \$.25x + \$.40y$

Constraints:

$$y \le 60$$
$$5x + 2y \le 500$$
$$.25x + .5y \le 40$$
$$x \ge 0$$
$$y \ge 0$$

Sofa Factories

A company makes two types of sofas, regular and long, at two locations, one in Hickory and one in Lenoir. The plant in Hickory has a daily operating budget of $45,000 and can produce at most 300 sofas daily in any combination. It costs $150 to make a regular sofa and $200 to make a long sofa at the Hickory plant. The Lenoir plant has a daily operating budget of $36,000, can produce at most 250 sofas daily in any combination and makes a regular sofa for $135 and a long sofa for $180. The company wants to limit production to a maximum of 250 regular sofas and 350 long sofas each day. If the company makes a profit of $50 on each regular sofa and $70 on each long sofa, how many of each type should be made at each plant in order to maximize profit? What is the maximum profit?

Sofa Factories

x = regular sofas made in Hickory
y = long sofas made in Hickory
z = regular sofas made in Lenoir
w = long sofas made in Lenoir

\$ constraint at Hickory: $150x + 200y \leq 45{,}000$
Hickory sofa limit: $x + y \leq 300$
\$ constraint at Lenoir: $135z + 180w \leq 36{,}000$
Lenoir sofa limit: $z + w \leq 250$
regular sofa limit: $x + z \leq 250$
long sofa limit: $y + w \leq 350$
non-neg: $x \geq 0,\ y \geq 0,\ z \geq 0,\ w \geq 0$

Maximize profit $P = 50x + 70y + 50z + 70w$